Answers for practice final exam
Math 165
Spring 2009

1. Compute the following limits:

(a) $\lim_{x \to -1} \frac{x^2 + x}{e^x - 1} = 0$

(b) $\lim_{x \to 0} \frac{x^2 + x}{e^x - 1} = 1$

2. Compute the derivatives of the following functions:

(a) $f(x) = \ln(e^x + 1)$ $\qquad f'(x) = \frac{e^x}{e^x + 1}$

(b) $g(x) = \frac{\sqrt{x+2}}{\tan x - 3}$ $\qquad g'(x) = \frac{\tan x - 3 - 2(x+2)\sec^2 x}{2\sqrt{x+2}(\tan x - 3)^2}$

(c) $h(x) = (x^2 + 1)^{(x^2+1)}$ $\qquad h'(x) = 2x[\ln(x^2+1) + 1](x^2+1)^{(x^2+1)}$

3. Compute the derivative of $f(x) = x^2 + x$, using the definition of the derivative. $\qquad f'(x) = 2x + 1$

4. Find a point where the tangent line to the curve $y = \cos x$ has slope 1/2.

Answer: $(-\pi/6, \sqrt{3}/2)$

5. The volume of a cube is increasing at a rate of $10\,\text{cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30 cm?

Answer: $4/3\,\text{cm}^2/\text{min}$

6. Find the relative maximum and relative minimum points of the function $f(x) = x^4 - x^3 + x^2 - 1$.

Relative minimum: $(0, -1)$
No relative maximum

7. A cone-shaped drinking cup is to be made to hold $27\,\mathrm{cm}^3$ of water. Find the height and radius of the cup that will use the smallest amount of paper.

radius: $r = \sqrt[6]{\dfrac{3^8}{2\pi^2}} = \dfrac{3\sqrt[3]{3\pi^2}\,\sqrt[6]{2^5}}{2\pi}$ cm

height: $h = \dfrac{81}{\pi\sqrt[3]{\frac{3^8}{2\pi^2}}} = \dfrac{3\sqrt[3]{6\pi^2}}{\pi}$ cm

8. Compute the following integrals:

(a) $\displaystyle\int_1^3 (x^2 + x)\,dx = \frac{38}{3}$

(b) $\displaystyle\int (\sqrt{x^3} + 1)^2\,dx = \frac{1}{4}x^4 + \frac{4}{5}x^{5/2} + x + C$

(c) $\displaystyle\int \sin(\sqrt{x^3} + 1)\sqrt{x}\,dx = -\frac{2}{3}\cos(\sqrt{x^3} + 1) + C$

(d) $\displaystyle\int \frac{1}{x\ln x}\,dx = \ln|\ln x| + C$

9. Find a number b such that the area under the curve $y = x/(1+x^2)$ above the interval $[0, b]$ is 1.

Answer: $b = \sqrt{e^2 - 1}$

Bonus. Compute $\displaystyle\int_1^3 (x^2 + x)\,dx$ using the definition of the definite integral.

Answer: $\displaystyle\int_1^3 (x^2 + x)\,dx = \frac{38}{3}$