Midterm 1 Practice Exam Answers
Math 165
Spring 2009

Note that I am only writing down the answers here. It will be important for you to show your steps on the actual exam.

1. Compute each limit:

(a) $\lim_{x \to 2} (x^2 - 3e^x) = 4 - 3e^2$

(b) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 + x - 6} = \frac{4}{5}$

(c) $\lim_{x \to \infty} \frac{\sqrt[3]{8x^6 + 1}}{3x^2 + x} = \frac{2}{3}$

(d) $\lim_{x \to 0} x \cot(3x) = \frac{1}{3}$ (use $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$)

(e) $\lim_{x \to 0} x \sin(1/x) \cos(1/x) = 0$ (use Squeeze Theorem)

2. Use the definition of the derivative to find $f'(1)$ when $f(x) = x^3 - x$. $f'(1) = 2$.

3. Compute the derivative of each function:

(a) $f(x) = x^3 \tan^{-1} x$

$f'(x) = 3x^2 \tan^{-1} x + x^3 \frac{1}{1 + x^2} = x^2 \left(3 \tan^{-1} x + \frac{x}{1 + x^2} \right)$ (product rule)

(b) $g(x) = \sqrt{1 + \sqrt{x}}$

$g'(x) = \frac{1}{4\sqrt{x(1 + \sqrt{x})}}$ (chain rule)

(c) $h(x) = \frac{\ln(x^4 + 1)}{e^{\tan x}}$

$h'(x) = \frac{1 - (x_1^4) \ln(x^4 + 1) \sec^2 x}{(x^4 + 1) e^{\tan x}}$ (quotient rule and chain rule)

(d) $k(x) = x^{\cos x}$ (logarithmic differentiation)

$$k'(x) = x^{\cos x}\left[\frac{1}{x}\cos x - \sin x \ln x\right] = \frac{x^{\cos x}\left[\cos x - x\sin x \ln x\right]}{x}$$

4. Find dy/dx when $x^6 + x^3y^2 + y^6 = 1$. (implicit differentiation)

$$\frac{dy}{dx} = -\frac{6x^5 + 3x^2y^2}{2x^3y + 6y^5} = -\frac{3x^2(2x^3 + y^2)}{2y(x^3 + 3y^4)}$$

5. Find the points where the tangent line to the curve $y = x^3 - 2x^2 + 1$ is horizontal.

$(1, 0)$ and $(4/3, -5/27)$

Bonus. Use the precise definition of the limit to prove the Sum Law: If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}[f(x) + g(x)] = L + M$.

See p. 115 of the text.