MATH 721, Algebra II
Exercises 9 and 10
Due Fri 28 Mar

Exercise 1. Let $K \to L$ be a field extension, and let $u, v \in L$ be algebraic over K. Show that, if the minimal polynomials of u and v over K have relatively prime degrees, then the minimal polynomial of u is irreducible in $K(v)[x]$.

Exercise 2. Let K be a splitting field for the polynomial $g = x^3 - 2$ over $\mathbb{Q}$. Find an element $u \in K$ such that $K = \mathbb{Q}(u)$.

Exercise 3. Find the Galois groups of the following field extensions:
(a) $\mathbb{Q} \subseteq K$ where K is a splitting field for the polynomial $f = x^2 - 2$,
(b) $\mathbb{Q} \subseteq L$ where L is a splitting field for the polynomial $g = x^3 - 2$,
(c) $\mathbb{Q} \subseteq M$ where M is a splitting field for the polynomial $h = (x^3 - 2)(x^2 - 2)$.
(d) $\mathbb{Q} \subseteq N$ where N is a splitting field for the polynomial $h = x^5 - 7$.

Exercise 4. Show that $\sqrt{2} + \sqrt{3}$ is algebraic over $\mathbb{Q}$ of degree 4.

Exercise 5. Let $p_1, \ldots, p_n$ be distinct positive prime integers. Show that the extension $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{p_1}, \ldots, \sqrt{p_n})$ is Galois and has degree 2^n.