MATH 721, Algebra II
Exercises 8
Due Fri 14 Mar

Exercise 1. Let K be a field.

(a) Show that $\mathrm{char}(K) = 0$ if and only if there is a homomorphism of fields $\mathbb{Q} \to K$.
(b) Show that $\mathrm{char}(K) = p > 0$ if and only if there is a homomorphism of fields $\mathbb{Z}/(p) \to K$.

Exercise 2. (Freshman dream) Let K be a field with $\mathrm{char}(K) = p > 0$. Show that, for all $a, b \in K$ we have $(a + b)^{p^n} = a^{p^n} + b^{p^n}$ for each integer $n \geqslant 1$.

Exercise 3. For each polynomial $f \in \mathbb{Q}[x]$ from the following list, let L be a splitting field for f over $\mathbb{Q}$ and find $[L : \mathbb{Q}]$:
$x^2 - 2$, $x^3 - 2$, $(x^2 - 2)(x^3 - 2)$ and $x^5 - 7$.