

MATH 721, Algebra II

Exercises 5

Due Fri 15 Feb

Exercise 1. Let $K \subseteq L$ be a field extension and let $X \subseteq L$. Show that there is an isomorphism of fields $K(X) \cong \mathbb{Q}(K[X])$.

Exercise 2. Let K be a field. Let $n \geq 1$ and let $K(x_1, \dots, x_n)$ be the field of rational functions over K in n variables. Show that each $u \in K(x_1, \dots, x_n) - K$ is transcendental over K .

Exercise 3. Let $K \subseteq L$ be a field extension and let $u \in L$. Show that, if u is algebraic over K with odd degree, then u^2 is algebraic over K with odd degree, and $K(u^2) = K(u)$.

Exercise 4. Let $K \subseteq L$ be a field extension and let $u \in L$. Let $K[x]$ be the polynomial ring in one variable over K . Assume that $x^n - a \in K[x]$ is irreducible and u is a root of $x^n - a$, and let $m \mid n$. Show that the degree of u^m over K is n/m . What is the irreducible polynomial for u^m over K ?