MATH 720, Algebra I
Exercises 11
Due Fri 09 Nov

Exercise 1. Let R be a ring with identity, and let $X_1, \ldots, X_n \subseteq R$ be subsets.

(a) Show that $\sum_{j=1}^{n}(X_j) = (\cup_{j=1}^{n} X_j)$.

(b) Assume that R is also commutative. Set

$$X = \{x_1 \cdots x_n \mid x_j \in X_j, j = 1, \ldots, n\}$$

and show $\prod_{j=1}^{n}(X_i) = (X)$.

Exercise 2. Let R be a ring, and let $I_1, \ldots, I_n \subseteq R$ be ideals.

(a) Show that $\prod_{j=1}^{n} I_j \subseteq \cap_{j=1}^{n} I_j$

(b) Give an example of a ring R with ideals I_1, I_2 such that $I_1 I_2 \neq I_1 \cap I_2$.

Exercise 3. Give an example of a ring R containing a maximal ideal that is not prime.