MATH 720, Algebra I
Exercises 8
Due Fri 12 Oct

Exercise 1. Let G be an abelian group and $K \leqslant G$. Show that the following conditions are equivalent.

(i) There exists a homomorphism $f\colon G \to K$ such that $f|_K = \mathrm{id}_K$;
(ii) There exists a subgroup $K' \leqslant G$ such that $G \cong K \oplus K'$.

Exercise 2. Let F be a free abelian group, and let $\pi\colon G \to F$ be an epimorphism of abelian groups. Show that there is a subgroup $K \leqslant G$ such that $G \cong K \oplus H$.

Exercise 3. Show that $\mathbb{Q}/\mathbb{Z}$ is a torsion abelian group, and find the p-torsion subgroup $(\mathbb{Q}/\mathbb{Z})_p$ for each prime p.

Exercise 4. How many abelian groups are there (up to isomorphism) of order 1728?