MATH 720, Algebra I
Exercises 2
Due Fri 07 Sep

Exercise 1. Let $f\colon G \to H$ be a group homomorphism.

(a) $\operatorname{Ker}(f) \leqslant G$ and $\operatorname{Im}(f) \leqslant H$.
(b) The function $g\colon G/\operatorname{Ker}(f) \to \operatorname{Im}(f)$ given by $\overline{g} \mapsto f(g)$ is a well-defined isomorphism and so $\operatorname{Im}(f) \cong G/\operatorname{Ker}(f)$.
(c) f is a monomorphism if and only if $\operatorname{Ker}(f) = \{e_G\}$.

Exercise 2. Let G be a group, and let $\{H_\lambda \mid \lambda \in \Lambda\}$ be a collection of subgroups of G.

(a) $\cap_\lambda H_\lambda \leqslant G$.
(b) If $H_\lambda \trianglelefteqslant G$ for all $\lambda \in \Lambda\}$, then $\cap_\lambda H_\lambda \trianglelefteqslant G$.
(c) Give an example of a group G with subgroups H and K such that $H \cup K$ is not a subgroup of G.

Exercise 3. Let G be a group and $H \trianglelefteqslant G$.

(a) If G is abelian, then G/H is abelian.
(b) Give an example where G/H is abelian and G is non-abelian.

Exercise 4. Show that the relation "$\cong$" is an equivalence relation.

Exercise 5. If $H \leqslant \mathbb{Z}$, then $H = n\mathbb{Z} = \langle n \rangle$ for some $n \geqslant 0$.

Exercise 6. If $H \leqslant G$ and $[G : H] = 2$, then $H \trianglelefteqslant G$.