MATH 720, Algebra I
Exercises 1
Due Fri 31 Aug

Exercise 1. Let G be a group and fix $a, b, c \in G$.

(a) If $ab = ac$, then $b = c$.
(b) If $ba = ca$, then $b = c$.
(c) The inverse of a in G is unique. [Assume that b and b' are inverses for a, and show $b = b'$.] [Once this is shown, we write a^{-1} for the unique inverse of a in G.]
(d) $(a^{-1})^{-1} = a$.
(e) $(ab)^{-1} = b^{-1}a^{-1}$.
(f) If $a^2 = a$, then $a = e$.

Exercise 2. Let G be a group. If $a^2 = e$ for all $a \in G$, then G is abelian.

Exercise 3. (Generalized Commutative Law) Let S be a commutative semigroup and fix elements $a_1, \ldots, a_n \in S$. For any permutation $i_1, \ldots, i_n$ of $1, \ldots, n$ show that $a_{i_1} \ldots a_{i_n} = a_1 \ldots a_n$.

Exercise 4. If $H \leqslant G$, then $e_H = e_G$, and for all $h \in H$ the inverse of h in H is the same as the inverse of h in G.

Exercise 5. Assume $H \trianglelefteq G$ and fix $a, b \in G$.

(a) The relation $\sim$ is an equivalence relation.
(b) The following conditions are equivalent:
    (i) $a \sim b$;
    (ii) $Ha = Hb$.
    (iii) $aH = bH$;